Space conversion by Affine transformation

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Abstract

This document summarizes space conversion between world, device, and image coordinates by Affine matrix.

Contents

1 BASICS

Affine transformation is a function between two Affine spaces which preserves points, straight lines and planes. Transformation of a coordinate (x, y) to another (x', y') is by Affine matrix A as

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = A \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{1}$$

or a set of coordinate X to another X' as

$$X' = AX. \tag{2}$$

The Affine matrix A can be estimated as

$$A = X'X^{-1}, \tag{3}$$

thus from known three pairs of coordinates in source and destination

$$\begin{pmatrix} x_1^{dst} & x_2^{dst} & x_3^{dst} \\ y_1^{dst} & y_2^{dst} & y_3^{dst} \\ 1 & 1 & 1 \end{pmatrix} = A \begin{pmatrix} x_1^{src} & x_2^{src} & x_3^{src} \\ y_1^{src} & y_2^{src} & y_3^{src} \\ 1 & 1 & 1 \end{pmatrix} \tag{4}$$

$$A = \begin{pmatrix} x_1^{dst} & x_2^{dst} & x_3^{dst} \\ y_1^{dst} & y_2^{dst} & y_3^{dst} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1^{src} & x_2^{src} & x_3^{src} \\ y_1^{src} & y_2^{src} & y_3^{src} \\ 1 & 1 & 1 \end{pmatrix}^{-1}. \tag{5}$$

where dst and src denote destination and source.

The Affine matrix is analyzed into slide, gain, distortion, and rotation such as

$$A = \begin{pmatrix} \multicolumn{2}{c}{\mathrm{T}} & \Delta x \\ & & \Delta y \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

$$T = gRD \tag{7}$$

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{8}$$

$$D = \begin{pmatrix} 1 & q_x \\ q_y & 1 \end{pmatrix} \tag{9}$$

where Δx, Δy, g, R, and D are slide in x and y, gain, and matrix of rotation and distortion. By $T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, gain ($g$), angle of rotation ($\theta$), and distortion ($q_x$, q_y) are described as $\theta = \tan^{-1} \frac{a-d}{c+b}$, $g = a\cos\theta - b\sin\theta$, $q_x = \frac{a/g-\cos\theta}{\sin\theta}$, and $q_y = \frac{c/g-\sin\theta}{\cos\theta}$.

2 STAGELIST AND IMAGE ON FUDO

Spaces where stagelist and image are on are shown in Table 1.

Table 1: List of spaces

space	remark
device	absolute in device
VS	absolute in world
xy-on-image	relative in image†

†Visualize spots in Medusa: origin is center of image and the longest side is 100

An image $X^{xy-on-image}$ is projected to device plane using Affine matrix *imajeoletry* A_{jeol} as

$$X^{dev} = A_{jeol} X^{xy-on-image}. \tag{10}$$

To convert stagelist or image from device to VS space, Affine matrix *stageometry* A_{stage} is involved as

$$X^{vs} = A_{stage} X^{dev}. \tag{11}$$

Based on Equation (10, 11), the image $X^{xy-on-image}$ is projected to VS plane by

$$X^{vs} = A_{stage} X^{dev} = A_{stage}(A_{jeol} X^{xy-on-image}) = A_{geo} X^{xy-on-image} \tag{12}$$

where A_{geo} ($\equiv A_{stage} A_{jeol}$) is referred as Affine matrix *imageometry*. Objects involved in space conversion is summarized in Table 2.

Table 2: Objects involved in space conversion

object	symbol	stored as	creator	remark
image	$X^{xy-on-image}$	image.jpg	JEOL JSM-7001F	—
imajeoletry	A_{jeol}	image.txt	JEOL JSM-7001F	—
stageometry	A_{stage}	stagelist@SEM@20160222.geo	vs-get-affine	—
	A_{stage}	stagelist@SEM@20160222.acp	Visual Stage 2007	—
stagelist	$X^{xy-on-image}$	image.pml	spots.m	—
	X^{dev}	stagelist@SEM@20160222.txt	spots-warp	—
	X^{vs}	stagelist.txt	Visual Stage 2007	—
imageometry	A_{geo} ($\equiv A_{stage} A_{jeol}$)	image.geo	image-get-affine	affine-xy2vs
	A_{geo} (interactive)	image.geo	vs-attach-image.m	affine-xy2vs

3 PRACTICE

By JEOL, "magnification" of image m_{SEM} is defined relative to 120,000 μm. When width of the image is to 100 μm, magnification is inferred as x1200. In that case g_{jeol} (g in A_{jeol}) is estimated to be $g_{jeol} = 100\ \mu\text{m}/100 = 1.0\ \mu$m. Conversion from well calibrated devices does not magnify space and g_{stage} (g in A_{stage}) is supposed to be unity, and g_{jeol} and g_{geo} (g in A_{geo}) will be the same. Example of these parameters are shown in Equations (13, 14) and Table 3.

$$g_{jeol} = \frac{1200}{m_{SEM}} \approx g_{geo} \tag{13}$$

$$m_{SEM} = \frac{1200}{g_{jeol}} \approx \frac{1200}{g_{geo}} \tag{14}$$

Table 3: Example of parameters with varying m_{SEM}, without rotation and distortion: See main text. $w^{xy-on-image}_{image}$ and w^{device}_{image} correspond to width of image in space of xy-on-image and device, respectively.

m_{SEM}	position	w^{dev}_{image}	$w^{xy-on-image}_{image}$	g_{jeol}	A_{jeol}	g_{stage}	A_{stage}	g_{geo}	A_{geo}
x1	$\begin{pmatrix} x^{dev}_a \\ y^{dev}_a \end{pmatrix}$	120000 μm	100	1200	$\begin{pmatrix} 1200 & 0 & x^{dev}_a \\ 0 & 1200 & y^{dev}_a \\ 0 & 0 & 1 \end{pmatrix}$	1	$\begin{pmatrix} 1 & 0 & \Delta x^{vs0}_{dev0} \\ 0 & 1 & \Delta y^{vs0}_{dev0} \\ 0 & 0 & 1 \end{pmatrix}$	1200	$\begin{pmatrix} 1200 & 0 & x^{vs}_a \\ 0 & 1200 & y^{vs}_a \\ 0 & 0 & 1 \end{pmatrix}$
x1000	$\begin{pmatrix} x^{dev}_b \\ y^{dev}_b \end{pmatrix}$	120 μm	100	1.2	$\begin{pmatrix} 1.2 & 0 & x^{dev}_b \\ 0 & 1.2 & y^{dev}_b \\ 0 & 0 & 1 \end{pmatrix}$	1	$\begin{pmatrix} 1 & 0 & \Delta x^{vs0}_{dev0} \\ 0 & 1 & \Delta y^{vs0}_{dev0} \\ 0 & 0 & 1 \end{pmatrix}$	1.2	$\begin{pmatrix} 1.2 & 0 & x^{vs}_b \\ 0 & 1.2 & y^{vs}_b \\ 0 & 0 & 1 \end{pmatrix}$
x1200	$\begin{pmatrix} x^{dev}_c \\ y^{dev}_c \end{pmatrix}$	100 μm	100	1	$\begin{pmatrix} 1 & 0 & x^{dev}_c \\ 0 & 1 & y^{dev}_c \\ 0 & 0 & 1 \end{pmatrix}$	1	$\begin{pmatrix} 1 & 0 & \Delta x^{vs0}_{dev0} \\ 0 & 1 & \Delta y^{vs0}_{dev0} \\ 0 & 0 & 1 \end{pmatrix}$	1	$\begin{pmatrix} 1 & 0 & x^{vs}_c \\ 0 & 1 & y^{vs}_c \\ 0 & 0 & 1 \end{pmatrix}$
x10000	$\begin{pmatrix} x^{dev}_d \\ y^{dev}_d \end{pmatrix}$	12 μm	100	0.12	$\begin{pmatrix} 0.12 & 0 & x^{dev}_d \\ 0 & 0.12 & y^{dev}_d \\ 0 & 0 & 1 \end{pmatrix}$	1	$\begin{pmatrix} 1 & 0 & \Delta x^{vs0}_{dev0} \\ 0 & 1 & \Delta y^{vs0}_{dev0} \\ 0 & 0 & 1 \end{pmatrix}$	0.12	$\begin{pmatrix} 0.12 & 0 & x^{vs}_d \\ 0 & 0.12 & y^{vs}_d \\ 0 & 0 & 1 \end{pmatrix}$

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